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MAGNETOHYDRODYNAMICS OF HEAVY FLUIDS

R. Kh. Zeitunyan

UDC 538.4

Four dimensionless parameters appear in the equations in connection with the discussion of the time-independent flow of an ideal compressible rotating plasma in a gravitational field: the Froude Fr, Rossby Ro, Mach M_0, and Alfvén A_0 numbers. Here it is assumed that A_0 and M_0 are simultaneously very small and satisfy the similarity relationship $A_0^2/M_0 = \nu_0$, where $\nu_0 = O(1)$ is a constant. First the case is analyzed in which $Fr \to 0$ and $A_0^2/Fr^2 = \lambda_0$, where $\lambda_0 = O(1)$ is a constant; the classical approximation of static equilibrium is obtained. If one notes that $Fr^2 = \gamma M_0^2/\beta_0$, where β_0 is the ratio of characteristic lengths, then it is necessary to discuss two cases. The first case corresponds to $\beta_0 = O(1)$, and a limiting system of equations is derived which permits studying atmospheric motions near the planets of the solar system, for which the characteristic angular rotational velocity is not very high ($A_0^2/Ro \ll 1$). The second case corresponds to $\beta_0 \to 0$ and $\beta_0/M_0 = \mu_0$, where $\mu_0 = O(1)$ is a new constant; it is possible to obtain a limiting system of equations which is suitable for analysis of the development of sunspots, where the magnetic and convective effects are closely linked.

1. Introduction

We will assume that only gravitational and electromagnetic forces are acting on the "fluid medium", which is treated as an ideal plasma (see [1] in connection with the definition of an ideal plasma). The equations which describe a nonsteady adiabatic flow of an infinitely conductive plasma rotating with angular velocity $\mathbf{\Omega}$ when viscosity and thermal conductivity are neglected have the form (the magnetic permeability μ is assumed to be constant):

$$\rho\{D\mathbf{v}/Dt + 2[\mathbf{\Omega}\cdot\mathbf{v}]\} + \nabla p + \rho g\mathbf{e}_3 = (1/\mu)[\mathrm{rot}\mathbf{B}\cdot\mathbf{B}]; \tag{1.1}$$

$$\partial\rho/\partial t + \mathrm{div}(\rho\mathbf{v}) = 0; \tag{1.2}$$

$$\mathrm{div}\,\mathbf{B} = 0; \tag{1.3}$$

$$\frac{DT}{Dt} - \frac{\gamma-1}{\gamma}\frac{T}{p}\frac{Dp}{Dt} = 0; \tag{1.4}$$

$$\partial\mathbf{B}/\partial t + \mathrm{rot}\,[\mathbf{B}\cdot\mathbf{v}] = 0. \tag{1.5}$$

The plasma is treated as an ideal gas with constant specific heats c_p and c_V ($\gamma = c_p/c_V$); therefore

Lille, France. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 37–42, March–April, 1980. Original article submitted March 20, 1979.

0021-8944/80/2102-0189 $07.50

$$p = R\rho T, \tag{1.6}$$

where $R = c_p(\gamma - 1)/\gamma$. Equations (1.1)-(1.6) form a closed system for the velocity vector $\mathbf{v}$ relative to the medium ($D/Dt = \partial/\partial t + (\mathbf{v}\cdot\nabla)$), the magnetic induction vector $\mathbf{B}$, and the scalars p (pressure), ρ (density), and T (temperature).

Equations (1.1)-(1.6) are written in a system of rectangular cartesian coordinates $Ox_1x_2x_3$; the $\mathbf{e}_i$ are unit vectors along the axes; and

$$\nabla \equiv \frac{\partial}{\partial x_i}\mathbf{e}_i.$$

Of course, it is necessary to supplement these equations with initial and boundary conditions; in particular, it would be necessary for the velocity to write the slipping condition along the wall. Concerning the magnetic field, since the electrical conductivity is assumed to be infinite, it is necessary to set up conditions for $\mathbf{B}$ similar to the conditions for $\mathbf{v}$. Some limiting forms of the system (1.1)-(1.6) are derived in this paper, and dimensionless parameters are exhibited which determine flows which satisfy Eqs. (1.1)-(1.6).

2. The Reduced Equations

Equation (1.1) contains five dimensionless parameters, which permit assessing the relative importance of different effects. We will make the replacement of variables:

$$\mathbf{v} = U_0\mathbf{u},\ \mathbf{B} = B_0\mathbf{b},\ \mathbf{x} = L_0\boldsymbol{\xi},\ t = t_0\tau,$$
$$p = p_0p',\ T = T_0T',\ \rho = \rho_0\rho',\ \boldsymbol{\Omega} = \Omega_0\boldsymbol{\omega},$$

where U_0, B_0, L_0, t_0, p_0, T_0, ρ_0, and Ω_0 are characteristic scalar quantities. Then one can write Eq. (1.1) in the form

$$\rho'\left\{\mathrm{St}\frac{\partial\mathbf{u}}{\partial\tau} + (\mathbf{u}\nabla)\mathbf{u} + \frac{1}{\mathrm{Ro}}[\boldsymbol{\omega}\cdot\mathbf{u}]\right\} + \frac{1}{\gamma\mathrm{M}_0^2}\nabla p' + \frac{1}{\mathrm{Fr}^2}\rho'\mathbf{e}_3 = \frac{1}{\mathrm{A}_0^2}[\mathrm{rot}\,\mathbf{b}\cdot\mathbf{b}], \tag{2.1}$$

where the dimensionless parameters

$$\mathrm{St} = L_0/t_0U_0,\ \mathrm{Ro} = U_0/2\Omega_0L_0,\ \mathrm{M}_0 = U_0/(\gamma RT_0)^{1/2},$$
$$\mathrm{Fr} = U_0/(gL_0)^{1/2},\ \mathrm{A}_0 = U_0/[B_0/(\mu\rho_0)^{1/2}]$$

are introduced, which are the Strouhal, Rossby, Mach, Froude, and Alfvén numbers, respectively.

The case $\mathrm{Fr} = \infty$ corresponds to classical magnetohydrodynamics without the gravitational force taken into account. When $\mathrm{Fr} \neq \infty$, it is more convenient to conduct the analysis by using perturbations of thermodynamic quantities [2]:

$$\pi = (p - p_\infty)/p_\infty,\ \Theta = (T - T_\infty)/T_\infty,\ \sigma = (\rho - \rho_\infty)/\rho_\infty,$$

where p_∞, ρ_∞, T_∞ are generally functions of the vertical coordinate x_3, characterize the thermodynamic standard state, and satisfy the relationships

$$p_\infty = R\rho_\infty T_\infty,\ dp_\infty/dx_3 + \rho_\infty g = 0,\ -dT_\infty/dx_3 = \Gamma_\infty \tag{2.2}$$

(the quantity Γ_∞ is assumed to be known).

It should be noted that the standard state (2.2) is in agreement with the system of equations (1.1)-(1.6) if, in particular,

$$\mathbf{v} \equiv \mathbf{v}_\infty^0 = U_\infty^0\mathbf{e}_1 + V_\infty^0\mathbf{e}_2$$

is a constant velocity vector and $\mathbf{B} \equiv \mathbf{B}_\infty$ is a harmonic vector, i.e., $\mathrm{rot}\,\mathbf{B}_\infty = 0$ and $\mathrm{div}\,\mathbf{B}_\infty = 0$, which is perpendicular to $\mathbf{v}_\infty^0$ ($\mathbf{v}_\infty^0\cdot\mathbf{B}_\infty = 0$) and $\Omega_0 = 0$. Thus if $\Omega_0 \equiv 0$ ($\mathrm{Ro} \equiv \infty$) and one sets $\Gamma_\infty \equiv \Gamma_\infty^0 = \mathrm{const}$, then one can write in place of Eqs. (1.1)-(1.6) the following dimensionless equations:

$$(1+\sigma)\left\{\mathrm{St}\frac{\partial\mathbf{u}}{\partial\tau} + (\mathbf{u}\cdot\nabla)\mathbf{u}\right\} + \frac{T'_\infty}{\gamma\mathrm{M}_0^2}\nabla\pi - \frac{1}{\mathrm{Fr}^2}(1+\sigma)\Theta\mathbf{e}_3 = \frac{1}{\mathrm{A}_0^2}[\mathrm{rot}\,\mathbf{b}\cdot\mathbf{b}],$$
$$\mathrm{St}\frac{\partial\sigma}{\partial\tau} + \mathbf{u}\cdot\nabla\sigma + (1+\sigma)\left[(\alpha_0 - \beta_0)\frac{u_3}{T'_\infty} + \mathrm{div}\,\mathbf{u}\right] = 0,$$
$$(1+\sigma)\left\{\mathrm{St}\frac{\partial\Theta}{\partial\tau} + \mathbf{u}\cdot\nabla\Theta\right\} - \frac{\gamma-1}{\gamma}\left\{\mathrm{St}\frac{\partial\pi}{\partial t} + \mathbf{u}\cdot\nabla\pi\right\} + (1+\pi)\left[\frac{\gamma-1}{\gamma}\beta_0 - \alpha_0\right]\frac{u_3}{T'_\infty} = 0, \tag{2.3}$$
$$\mathrm{St}\frac{\partial\mathbf{b}}{\partial\tau} + \mathrm{rot}\,[\mathbf{b}\cdot\mathbf{u}] = 0,\qquad \mathrm{div}\,\mathbf{b} = 0,\qquad \pi = \sigma + \Theta + \sigma\Theta,$$

where two new dimensionless parameters

$$\alpha_0 = \frac{\Gamma^0_\infty L_0}{T_0}, \qquad \beta_0 = \frac{gL_0}{RT_0} \equiv \frac{\gamma M_0^2}{Fr^2}, \tag{2.4}$$

are introduced which characterize the standard state.

3. The Case Fr $\to 0$

Let us consider Eq. (2.1) on the assumption of quasisteadiness (St $\equiv 0$). We will assume that the Alfvén number A_0 is infinitely small ($B_0 \gg \sqrt{\mu\rho_0}U_0$), and we will derive the limiting form of the equation

$$\rho'\left\{A_0^2(\mathbf{u}\cdot\nabla)\,\mathbf{u} + \frac{A_0^2}{Ro}[\boldsymbol{\omega}\cdot\mathbf{u}]\right\} + \frac{1}{\gamma M_0}\frac{A_0^2}{M_0}\nabla p' + \left(A_0^2/Fr^2\right)\rho'\mathbf{e}_3 = [\operatorname{rot}\mathbf{b}\cdot\mathbf{b}], \tag{3.1}$$

when $A_0 \to 0$ with ξ fixed.

In order to derive the supporting equation from (3.1), it is necessary that M_0 or Ro also tend to zero. Assuming in addition that Fr $\to 0$, we find the following limiting equation:

$$\varkappa_0[\boldsymbol{\omega}\cdot\mathbf{u}_0] + \frac{\nu_0}{\gamma}\nabla p_1' + \lambda_0\mathbf{e}_3 = [\operatorname{rot}\mathbf{b}_0\cdot\mathbf{b}_0], \tag{3.2}$$

if one assumes the existence of the similarity relationships

$$\frac{A_0^2}{Ro} = \varkappa_0, \quad \frac{A_0^2}{M_0} = \nu_0, \quad \frac{A_0^2}{Fr^2} = \lambda_0 \tag{3.3}$$

and seeks the solution of Eq. (3.1) in the form of the asymptotic expansions

$$\mathbf{u} = \mathbf{u}_0 + o(1), \quad \mathbf{b} = \mathbf{b}_0 + o(1), \quad p' = 1 + M_0 p_1' + o(M_0),$$

$$\rho = 1 + M_0\rho_1' + o(M_0).$$

Setting

$$T' = 1 + M_0 T_1' + o(M_0),$$

one can combine the limiting equation (3.2) with the following equations:

$$\operatorname{div}\mathbf{u}_0 = 0, \ \operatorname{div}\mathbf{b}_0 = 0; \tag{3.4}$$

$$\operatorname{rot}[\mathbf{b}_0\cdot\mathbf{u}_0] = 0, \quad \mathbf{u}_0\cdot\nabla T_1' = \frac{\gamma-1}{\gamma}\mathbf{u}_0\cdot\nabla p_1', \quad \rho_1' = p_1' - T_1'. \tag{3.5}$$

We note that the limiting system (3.2), (3.4), and (3.5) remains valid on the assumption

$$\frac{RT_0}{g} \gg L_0 \gg \frac{U_0^2}{g},$$

because β_0 should tend to zero when $M_0 \to 0$ according to the relationship $\beta_0 = \gamma\frac{\lambda_0}{\nu_0}M_0$ which makes the last two relationships in (3.3) and the second one from (2.4) independent. In a more particular case ($\varkappa_0 \equiv 0$) the limiting system (3.2), (3.4), and (3.5) decomposes into three subsystems:

$$[\operatorname{rot}\mathbf{b}_0\cdot\mathbf{b}_0] = \nabla P, \quad \operatorname{div}\mathbf{b}_0 = 0, \quad P \equiv \frac{\nu_0}{\gamma}p_1' + \lambda_0\xi_3; \tag{3.6}$$

$$\operatorname{rot}[\mathbf{b}_0\cdot\mathbf{u}_0] = 0, \ \operatorname{div}\mathbf{u}_0 = 0; \tag{3.7}$$

$$\mathbf{u}_0\cdot\nabla T_1' = \frac{\gamma-1}{\gamma}\nabla p_1', \quad \rho_1' = p_1' - T_1'. \tag{3.8}$$

The subsystem (3.6), which we will call "static equilibrium," has been analyzed in particular in [3]; this subsystem permits determining $\mathbf{b}_0$ and p_1' with the appropriate boundary conditions. We note that if one introduces the two scalar potentials ψ and χ according to the conditions

$$\operatorname{div}\mathbf{b}_0 = 0 \Rightarrow \mathbf{b}_0 = [\nabla\psi\cdot\nabla\chi],$$

then the first of Eqs. (3.6) gives

$$\mathbf{b}_0\cdot\nabla P = 0 \Rightarrow P = \Pi(\psi, \chi)$$

and the limiting form (3.6) is equivalent to a system of two first integrals (see [4] and [2])

$$\operatorname{rot}\ \mathbf{b}_0\cdot\nabla\psi = \partial\Pi/\partial\chi, \ \operatorname{rot}\ \mathbf{b}_0\cdot\nabla\chi = -\partial\Pi/\partial\psi.$$

The function $\Pi(\psi, \chi)$ is determined for continuous flows with the help of the boundary conditions. As soon as the values of $\mathbf{b}_0$ and p_1' are found, one can calculate $\mathbf{u}_0$ from the system (3.7) of linear equations in first-order partial derivatives with respect to ξ, and then one can find T_1 and ρ_1 from (3.8).

4. The Case $\beta_0 = O(1)$

In order to clearly show the effect of a gravitational field, it is convenient to use the system of equations (2.3). We will consider the case $\beta_0 = O(1)$, i.e., L_0 is of the order of RT_0/g. In addition let $A_0 \to 0$, so that $A_0^2/M_0 = \nu_0$. We will seek a solution of Eqs. (2.3) in the form of the following asymptotic expansions:

$$\mathbf{u} = \mathbf{u}_0 + o(1),\ \mathbf{b} = \mathbf{b}_0 + o(1),\ \pi = M_0\pi_1 + o(M_0),$$
$$\sigma = M_0\sigma_1 + o(M_0),\ \Theta = M_0\Theta_1 + o(M_0). \tag{4.1}$$

If the Strouhal number St is fixed (or is identically equal to zero), we obtain a strong degeneracy in the zeroth order (when $M_0 \to 0$), since $u_{3,0} \equiv 0$ follows from the third equation of the system (2.3) if one only assumes that the parameter α_0 introduced by the first of Eqs. (2.4) is not fixed but satisfies the condition

$$\alpha_0 = \frac{\gamma - 1}{\gamma}\beta_0 - k_0 M_0, \tag{4.2}$$

where k_0 is a constant similarity parameter. With the restriction (4.2) the limiting system of zeroth-order equations, which "decouples" Eqs. (2.3) with Eqs. (4.1) taken into account, takes on the following form if one sets $\nu_0 \neq 0$ and $St \equiv 0$:

$$\pi_1 = \omega_1 + \Theta_1,\quad \operatorname{div}\mathbf{u}_0 = \frac{\beta_0}{\gamma T'_\infty}u_{3,0},\quad \operatorname{div}\mathbf{b}_0 = 0,$$
$$[\operatorname{rot}\mathbf{b}_0\cdot\mathbf{b}_0] = \frac{\nu_0}{\gamma}\{T'_\infty\nabla\pi_1 - \beta_0\Theta_1\mathbf{e}_3\}, \tag{4.3}$$
$$\mathbf{u}_0\cdot\nabla\left\{\Theta_1 - \frac{\gamma-1}{\gamma}\pi_1\right\} + \frac{k_0}{T'_\infty}u_{3,0} = 0,\quad \operatorname{rot}[\mathbf{b}_0\cdot\mathbf{u}_0] = 0,$$

where $T'_\infty = 1 - \frac{\gamma-1}{\gamma}\beta_0\xi_3;\ T_\infty(0) \equiv T_0$. The system (4.3) is strongly coupled and describes magnetoconvective motion in relatively thick layers; the thicker the layer is, the weaker the gravitational field is and the higher the standard temperature on the surface of the earth is.

The system (4.3) may be of interest, in particular, for the study of flows in the atmospheres of planets of the solar system [5]. It should be noted that the limiting equations (4.3) are applicable to the study of the atmospheres of planets whose characteristic angular rotational velocity Ω_0 satisfies the inequality

$$\Omega_0 \ll \frac{gB_0^2}{2R\mu\rho_0U_0T_0}.$$

5. The Case $\beta_0 \to 0$

Now let us assume that $\beta_0 \to 0$ and $M_0 \to 0$ in Eqs. (2.3), so that

$$\beta_0/M_0 = \mu_0,$$

where μ_0 is a constant similarity parameter. Then it is necessary to seek the solution of Eqs. (2.3) in the form

$$\mathbf{u} = \mathbf{u}_0 + o(1),\ \mathbf{b} = \mathbf{b}_0 + o(1),\ \sigma = \sigma_0 + o(1),$$
$$\Theta = \Theta_0 + o(1),\ \pi = M_0\pi_1 + o(M_0).$$

When $St \equiv 0$, we obtain the following limiting equation in the zeroth order by taking into account the second of Eqs. (3.3) (with ν_0 as the second constant similarity parameter):

$$[\operatorname{rot}\mathbf{b}_0\cdot\mathbf{b}_0] = \frac{\nu_0}{\gamma}\{T'_\infty\nabla\pi_1 + \mu_0\sigma_0\mathbf{e}_3\},$$
$$\mathbf{u}_0\cdot\nabla\sigma_0 + (1+\sigma_0)\left\{\alpha_0\frac{u_{3,0}}{T'_\infty} + \operatorname{div}\mathbf{u}_0\right\} = 0, \tag{5.1}$$
$$(1+\sigma_0)\,\mathbf{u}_0\cdot\nabla\Theta_0 = \frac{\alpha_0}{T'_\infty}u_{3,0},\quad \operatorname{rot}[\mathbf{b}_0\cdot\mathbf{u}_0] = 0,\quad \operatorname{div}\mathbf{b}_0 = 0,\quad \Theta_0(1+\sigma_0) = -\sigma_0.$$

The parameter α_0 in the system (5.1) is fixed; if one assumes that $\alpha_0 \to 0$, then we obtain a new limiting system in place of Eqs. (5.1):

$$\mathbf{u}_0\cdot\nabla\sigma_0 = 0,\ \operatorname{div}\mathbf{u}_0 = 0,$$
$$[\operatorname{rot}\mathbf{b}_0\cdot\mathbf{b}_0] = \frac{\nu_0}{\gamma}(\nabla\pi_1 + \mu_0\sigma_0\mathbf{e}_3), \tag{5.2}$$
$$\operatorname{div}\mathbf{b}_0 = 0,\ \operatorname{rot}[\mathbf{b}_0\cdot\mathbf{u}_0] = 0,\ \Theta_0 = -\sigma_0/(1+\sigma_0).$$

The limiting system corresponding to the Boussinesq approximation for small Rossby numbers is reduced to the form (5.2) (see Ref. 6). The system (5.2) may be of interest in connection with the investigation of the formation of sunspots, where magnetic and convective effects are coupled.

The theory of magnetohydrodynamic flow of a heavy fluid at small Alfvén numbers which has been outlined above is similar from the conceptual standpoint to the theory of the flow of a heavy rotating fluid at small Rossby numbers. There is also a great analogy between the static equilibrium approximation (3.6) discussed in Sec. 3 and the classical quasigeostrophic approximation in meteorology [6].

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SYMMETRIC COLLISION OF TWO-LAYER JETS OF AN IDEAL INCOMPRESSIBLE LIQUID

S. A. Kinelovskii and Yu. A. Trishin

UDC 532.522

1. We consider the problem of finding the potential flow arising after the symmetric collision of plane two-layer free jets of an ideal incompressible liquid. Assuming that the flow is steady state, we shall analyze the conditions that must be satisfied in this case by the flows in the different layers of the colliding jets. For simplicity, by virtue of the symmetry, we can replace the plane of symmetry with a rigid stationary wall and consider the stationary problem of a two-layer jet of an ideal incompressible liquid hitting this wall. The flow in each of the layers of the jet is characterized by its value of the Bernoulli integral constant. Assuming that the pressure at infinity and on the free streamlines is zero, we denote by h the ratio of the Bernoulli integral constants in the layers;

$$h=\frac{\frac{1}{2}\rho_1 v_1^2}{\frac{1}{2}\rho_2 v_2^2}, \tag{1.1}$$

where v_1 and v_2 are the liquid velocities in the layer at infinity, and the subscripts 1 and 2 are assigned, respectively, to the external (outside the wall) and internal layers of the two-layer jet. In the general case the densities of the layers, ρ_1 and ρ_2, and the velocities, v_1 and v_2, are different. In addition, the problem also depends on the geometric parameters specified at infinity, such as the thicknesses of the layers and the angle of inclination of the jet to the wall. Depending on the values of all these parameters, it is possible in principle to have three variants of the flow arising when a two-layer jet hits the wall; a) The forward jet (the pestle) is inhomogeneous, while the return jet, (the cumulative jet) is homogeneous; b) the pestle and the cumulative jet are homogeneous; c) the pestle is homogeneous, while the cumulative jet is inhomogeneous.

Figure 1 shows the flow configuration corresponding to condition a), with a homogeneous jet and an inhomogeneous pestle, where ρ_1 is the density of the liquid layer external to the wall, ρ_2 is the density of the liquid layer inside the wall, and their velocities at infinity are v_1 and v_2; δ_1 and δ_2 are the thicknesses of the layers of the incident jet at the point at infinity, B; δ_1 is the thickness of the external layer of the pestle at the

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 42-52, March-April, 1980. Original article submitted June 5, 1979.